The Inflection-Scaling Method: A new method for calculating J_c in trouser tear specimens in presence of remote energy absorption without optical observation of crack initiation

Byung Kim, Michael McKinley, William Vogt

Mechanical and Industrial Engineering Department, University of Massachusetts Amherst, MA 01003, USA.

Abstract

The Inflection-Scaling Method is presented as a new means for calculating the critical J-integral value J_c in trouser tear specimens. This method calculates crack initiation points of trouser tear specimens by utilizing the first derivative of the load-displacement curve to locate inflection points. Remote energy absorption can be accounted for by applying a scaling factor based on the value of the slope profiles at inflection points. This method is particularly promising when specimens have undergone significant surface deterioration that would otherwise impede observation of crack initiation. Consistency in J_c values taken from both the J-integral and construction of an R-curve was improved over conventional methods. The Inflection-Scaling Method eliminates the need for optical observation of crack initiation and bypasses error in leg length due to specimen installation.

Keywords: Inflection-Scaling Method; J-integral; trouser tear; crack initiation; remote energy; locus method; R-curve

1. Introduction

The energy required for crack initiation, J_c, is an important failure criterion for cracking [1]. Using the methods outlined in [2], the critical value of the *J*-integral, J_c, can be determined through analysis of the load-displacement record of a specimen failing by crack propagation in tension. By creating a locus line of crack initiation points on the load-displacement plot, J_c can be determined by the following equation:

$$J_c = \frac{-1}{B} \frac{\Delta U_c(a)}{\Delta a}$$

where B is specimen thickness, a is specimen initial crack length. $U_c(a)$ is the area enclosed between the locus line, the load curve corresponding to a, and the displacement axis [3]. In plotting $U_c(a)/B$ vs. a, the slope of the fitted linear line may be interpreted as J_c.

The resistance curve, or simply *R*-curve, has been reported to be a useful method of characterizing the change in resistance to crack propagation as a function of crack growth [4, 5]. As shown in [4], an *R*-curve can be constructed from the load-displacement curve of a trouser tear specimen. For a given material, the locus of crack initiation points should be a horizontal line as the crack should initiate at the same load regardless of trouser leg length. This allows J_c to be rewritten as

$$J_c = \frac{1}{B} \frac{\text{AREA(OABC)}}{(a_4 - a_1)}$$

where AREA(OABC), shown in Figure 1, is the area enclosed by the locus line and the load curves corresponding to the smallest and largest initial crack lengths (a_1 and a_4 respectively).

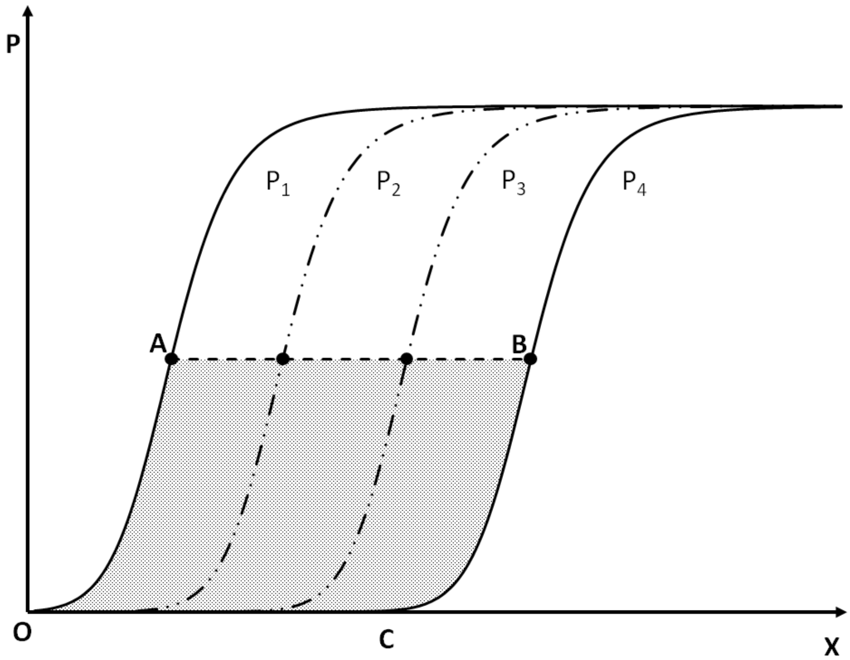


Figure 1-Schematic load-displacement curve of trouser tear specimens that only differ in initial crack length. Solid circles represent crack initiation points. The dashed line AB is the locus line of initiation. The shaded region OABC represents the change in energy required for crack initiation between P_1 and P_4.

It has been shown in [4] that in trouser tear tests, the resistance to crack propagation R can be written as

$$R = \frac{2P}{B} + \frac{2U_p}{B}$$

which is the sum of the energy associated with the applied load P and the remote strain energy U_p stored in one trouser leg of the specimen. If U_p can be shown to be negligible, then

$$R = \frac{2P}{B}$$

Thus, for a given load curve for a trouser tear test, the y-axis of the resistance curve is simply a scaling of the load curve by *2/B*.

To relate displacement *X* to specimen crack growth *C*, a technique utilizing linearization is employed as shown in [4, 5] where the crack growth *C* is given by

$$C = \frac{C_Q}{X_Q^{'}} \; X^{'}$$

where $X' = X - X_o$. X is displacement taken from the load curve and C_Q is given by:

$$C_Q = C_S - \frac{(X_S - X_Q)}{2}$$

where C_S is a known crack growth at point $S(X_S, P_S)$ in the steady state region of the load curve and $Q(X_Q, P_Q)$ is the point at which steady state crack propagation begins.

By plotting R vs. C an R-curve is constructed that has an initial value corresponding to J_c. Plotting the R-curve provides a second method of evaluating J_c, allowing comparison to values determined using Equation .

2. Inflection-Scaling Method for Determining J_c

2.1 Determining crack initiation from inflection points of load-displacement curves

The greatest source of error in the aforementioned methods is the subjectivity in observing crack initiation. Even after increasing resolution by using microscopes or reviewing video recordings of the initiation events, it is difficult to see precisely when the crack initiates at either the microscopic or macroscopic scale. Uncertainty in marking initiation points can result in large error in the calculation of J_c.

In a trouser tear test, the load increases to a critical value at which point energy is released in the form of crack initiation. At initiation, the first derivative of the load-displacement curve must be at a global maximum located at a point X_I which can be expressed as

$$M_i = \arg\max_x \frac{dP_i}{dx}$$

Using Equation , the crack initiation point is calculated for each load curve. A more accurate locus line can be generated using these calculated initiation points rather than observed initiation points. Subsequently, as compared to previously discussed methods, higher levels of accuracy and precision can be attained for the value of J_c and the construction of the *R*-curve.

2.2 Scaling Method for Remote Energy Compensation

Based on Equation , J_c is proportional to the enclosed area between the locus line of initiation and the load-displacement curves. However, there is substantial error in the load-displacement curves stemming from the remote strain energy stored in the trouser legs. A current method described in the literature [4] reduces error due to remote energy storage by shifting the load-displacement curve of the shortest specimen. The longer legs of the specimen with crack length a_4 store more strain energy, resulting in a lower energy release rate.

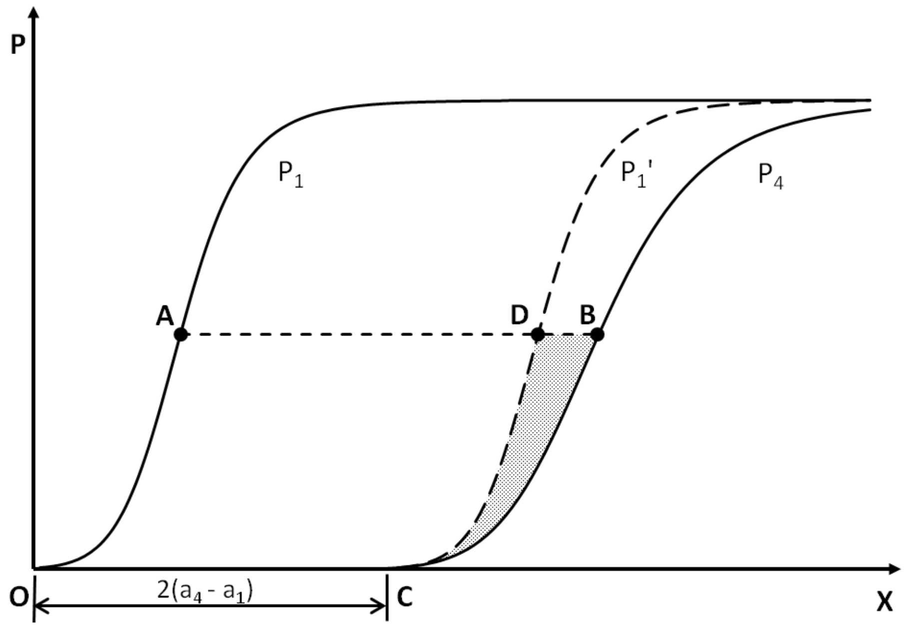


Figure 2- Shift of curve P_1 to account for remote energy storage in P_4 (the shaded region CDB). Dashed line ADB is the locus of initiation.

If there were no difference in remote energy storage between the longest and shortest specimens in a set, both load-displacement curves would be identical but the curve of the longest specimen would be a shifted version of the curve of the shortest specimen, as shown in Figure 2. [4] shows the value of the shift to be $2(a_4 - a_1)$.

Instead of shifting the curve of crack length a_1 to the right as a representation of the curve of crack length a_4, the curve of crack length a_4 can be scaled vertically such that the slope at each crack initiation point (points A and B in Figure 3) is equal. This scaling factor takes the form

$$V = \frac{\max \dfrac{dP_i}{dx}}{\max \dfrac{dP_f}{dx}}$$

where P_i and P_f are the load curves of the shortest and longest crack lengths, respectively. Of course, by basing this scaling action solely on the crack initiation point, the steady-state portion of the P_4 curve becomes meaningless. The remote energy absorption can be calculated as the area between the scaled curve, original curve, and the locus line. As in the shift method, subtracting this area from AREA(OABC) improves accuracy in determining J_c in the face of remote energy storage.

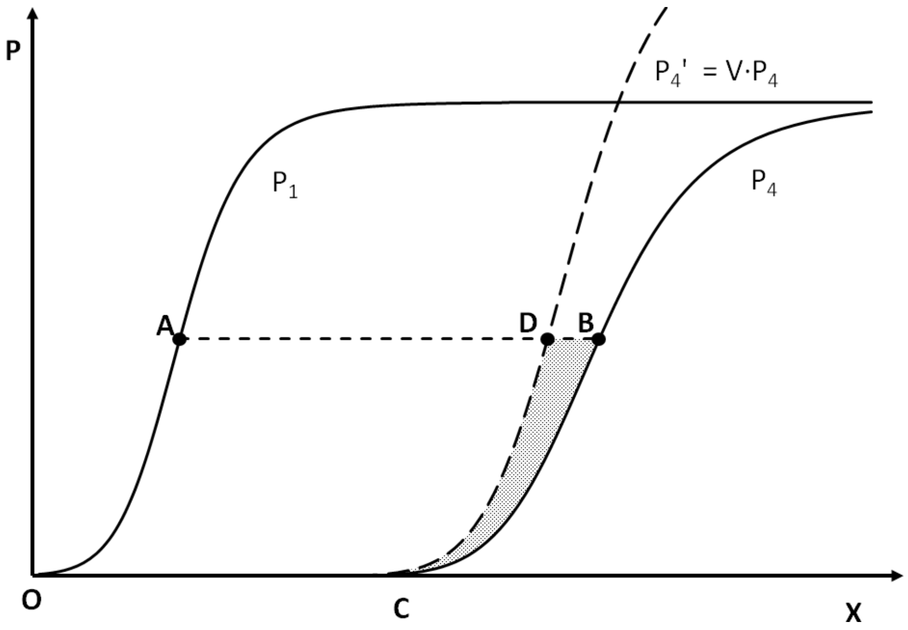


Figure 3 - Scaling of curve P_4 by V to account for remote energy storage (the shaded region CDB). Dashed line ADB is the locus of initiation.

When specimens are installed for testing, it is difficult to determine the exact length of the legs due to deformation caused by grip forces. The shift method can be greatly affected by this source of error. As seen in Figure 4, error in AREA(OADC) increases proportionally with error in leg length. The advantage of the proposed scaling method is that scaling the P_4 curve accounts for remote energy without relying on exact knowledge of specimen leg lengths.

Smoothing functions are employed in calculating the slope profiles of the load-displacement curves in order to eliminate the effect of noise. V is affected by the aggressiveness of the smoothing ratio. A larger smoothing ratio artificially lowers the peak amplitude of the curves but increases accuracy in peak position.

Conversely, a smaller smoothing ratio increases sensitivity to noise in peak position but does not distort peak amplitude. As shown in Equation , *V* is only dependent on the peak values of the derivative curves, not peak positions. However, there is some error as a result of smoothing, as shown in Figure 5. This smoothing error does not affect AREA(OADC) as significantly as the installation error seen in Figure 4.

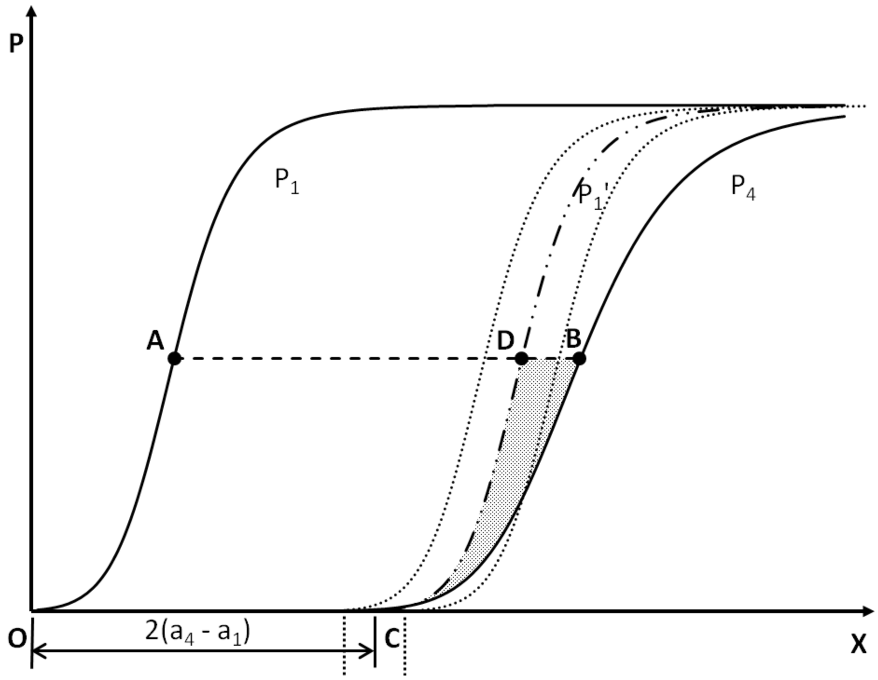


Figure 4 - Error in evaluating AREA(OADC) using the shift method. Curve P_1' is the shifted version of curve P_1. Dotted lines represent ± 10% error in leg length specimen P_1 leg length.

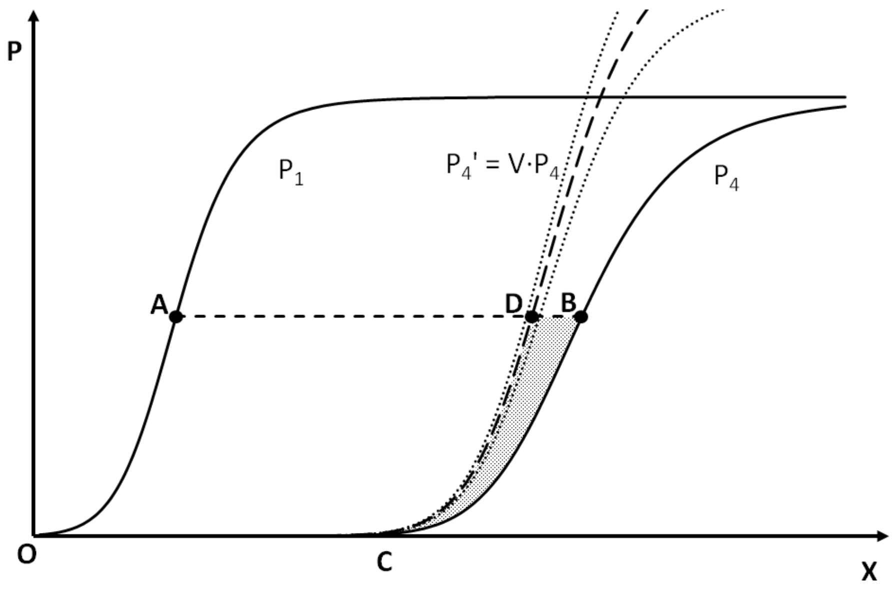


Figure 5 - Error in evaluating AREA(OADC) from scaling factor calculation. Curve P_4' is curve P_4 scaled by V. Dotted lines represent ± 10% error in V.

3. Experimental

For experimentation, extruded latex strips were fabricated from goggle straps produced by Malmsten, Ltd. This flexible material was chosen as it fails by cracking along the direction of its extrusion. Latex also provides an opportunity to test the robustness of the Inflection-Scaling Method in the face of material degradation through weathering. Each set was comprised of four specimens of width W, thickness B, and initial crack length a (see Figure 6).

Initial crack length [mm]	
a_1	2
a_2	4
a_3	8
a_4	10

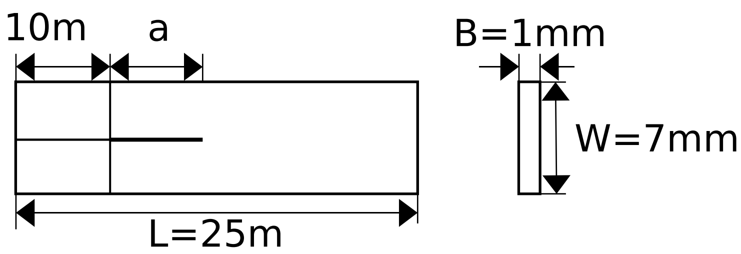


Figure 6 – Table of initial crack length values and drawing of trouser tear specimen. The shaded area denotes the grip region.

Trouser tears were performed at a average temperature of 70°F with a constant leg extension speed of 25 mm/min. Because latex is known to be susceptible to oxidation, weathered specimens were created by aging for approximately three months at 70°F.

Because it is difficult to distinguish between blunting and crack initiation, optical observation of initiation events can be inaccurate and somewhat subjective. To improve the accuracy of recording initiation points, a microscope was added to the test assembly. Each test was recorded with a camera attached to the microscope. This setup allowed for greater visual resolution and enabled review of initiation events.

4. Data Analysis

The inflection points and scaling factor were calculated by applying Equations and to the slope profiles shown in Figure 7. The raw experimental load-displacement curves are shown in Figure 8. From the data, it is evident that optical methods capture crack initiation events after a delay of about 1 second (for a crosshead speed of 25 mm/min) as the result of limited resolution. Initiation points obtained from weathered specimens were difficult to observe optically due to the unpredictable nature of crack initiation in these specimens.

Values of J_c were calculated using both the Inflection-Scaling Method and conventional methods. With each method, J_c was calculated in two ways: by utilizing Equation and by constructing an R-curve. The results of these calculations are presented in Table 1. It is clear that the Inflection-Scaling Method is capable of determining J_c with more precision than conventional methods regardless of how J_c is calculated. This method is particularly robust in the face of severe specimen degradation, where optical techniques are too imprecise.

Table 1 - Results for J_c using the proposed method for accounting for remote energy

Conventional Method	Unweathered	Weathered
J_c [kJ/m^2]	2.81	1.44
J_c from R-curve [kJ/m^2]	2.35	0.83
Percent error in J_c	16.4%	42.4%
Inflection-Scaling Method		
J_c [kJ/m^2]	2.28	1.64
J_c from R-curve [kJ/m^2]	2.12	1.65
Percent error in J_c	7.0%	0.6%

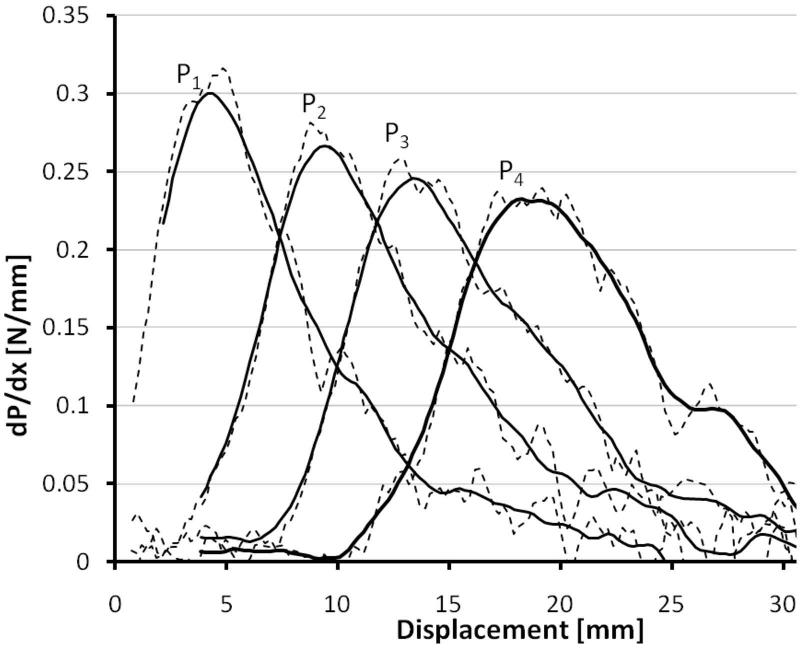
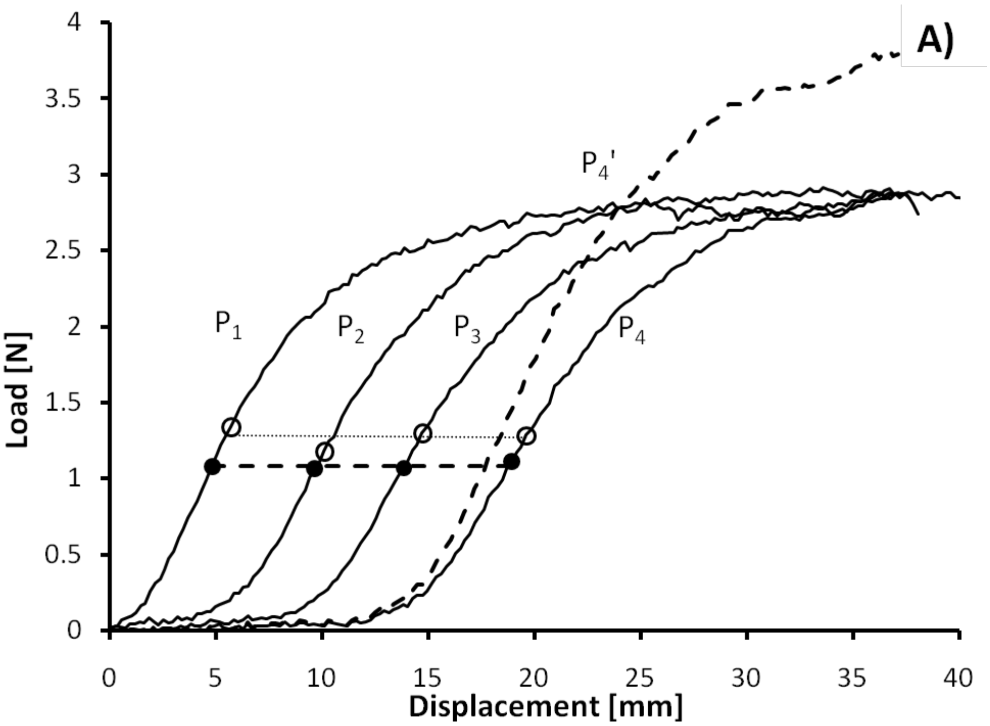


Figure 7 - Slope profile for load-displacement data. The dashed curves are slope profiles calculated using a 9-point stencil. The solid curves represent further filtering using a smoothing ratio of 0.15.


A)
Load [N]
Displacement [mm]
P1
P2
P3
P4
P4'
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20
25
30
35
40

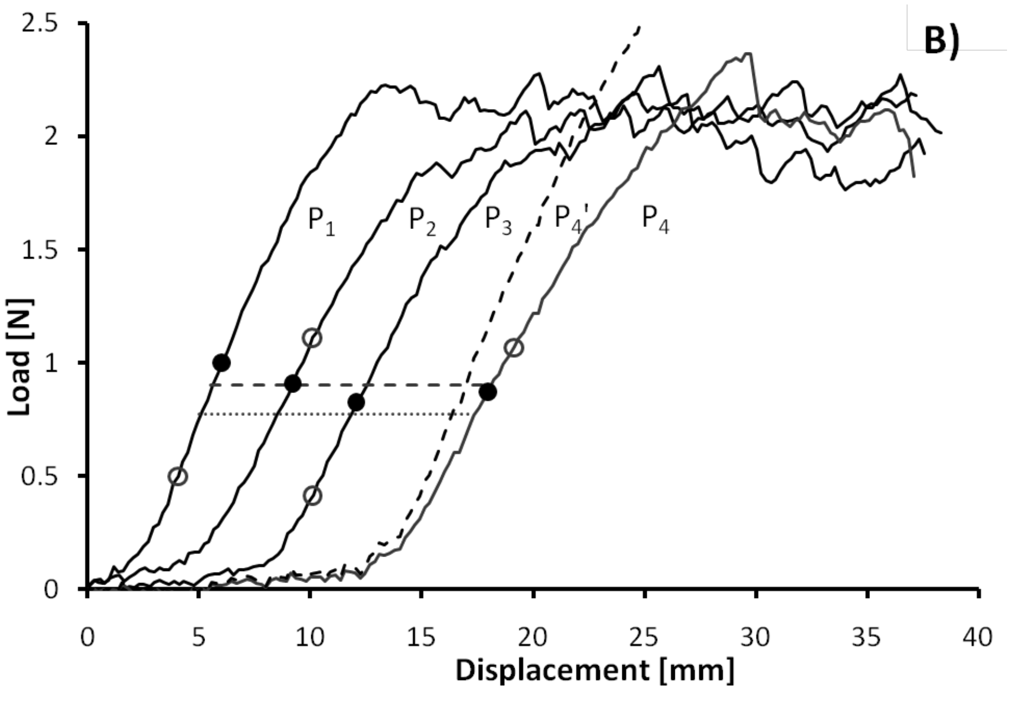

B)
Load [N]
Displacement [mm]
P1
P2
P3
P4'
P4
0
0.5
1
1.5
2
2.5
0
5
10
15
20
25
30
35
40

Figure 8 – Load-displacement records for A) unweathered and B) weathered trouser tear specimens where P_i corresponds to specimen with initial crack length a_i. Curve P_4^* is curve P_4 scaled by V. Open circles (with dotted locus line) represent optically observed crack initiation points whereas solid circles (with dashed locus line) represent calculated inflection points.

5. Conclusions

The Inflection-Scaling Method was proposed as a new means for calculating J_c in trouser tear specimens. It was shown that the first derivative of the load-displacement curve of trouser tear specimens can be used to locate crack initiation points on load-displacement curves. These slope profiles may also be used to account for remote energy absorption. Applying this method yielded increased consistency over conventional methods in J_c value as calculated by both construction of an R-curve and use of the J-integral. The Inflection-Scaling Method eliminates the need for optical observation of crack initiation and bypasses error in leg length due to specimen installation.

References

1. J.R. Rice, A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Journal of Applied Mechanics*, vol 35, pp. 379-386, 1968.

2. B. H. Kim and C. R. Joe, On the Resistance to Crack Initiation and Growth. *Engineering Fracture Mechanics*, vol 34, #1, pp. 221-231, 1989.

3. B. H. Kim and C. R. Joe, The Effect of Remote Energy Absorption in Determining Jc value. *Engineering Fracture Mechanics,* vol 32, #2, pp. 225-232, 1989.

4. C.R. Joe and B.H. Kim, A method to determine the R-curve of flexible materials using a monotonic load of a single trouser tear specimen. *International Journal of Fracture*, vol 44, pp. 15-26, 1990

5. B.H. Kim and M.A. Skolnick. On the Construction of an R-curve of Flexible Materials from the Load-displacement Record of a Trouser Tear. *International Journal of Fracture* vol 46 pp. R19-R24, 1990.